Eur. Phys. J. D **29**, 189–193 (2004)
DOI: 10.1140/epjd/e2004-00027-1

THE EUROPEAN
PHYSICAL JOURNAL D

Quantum-information processing in strong-excitation regime with trapped ions in microcavity

M. Feng[a]

Wuhan Institute of Physics and Mathematics, Chinese Academy of Sciences, Wuhan, 430071, P.R. China
and
School of Theoretical Physics and Department of Applied Physics, Hunan University, Changsha, 410082, P.R. China

Received 25 June 2003 / Received in final form 22 December 2003
Published online 2nd March 2004 –

Abstract. A quantum information processing scheme is proposed in a system with cold trapped ions embedded in a single mode microcavity in strong excitation regime. With suitable choice of frequencies of the laser and the cavity light as well as ion-laser coupling strength, multipartite entanglement would be generated among internal states of the ions, which is insensitive to decoherence due to the cavity decay and heating of the vibrational mode of the ions. As a practical example, the specific discussion is focused on the two-ion case. Some unique features of the Bell states are presented under the framework of our model, which would be useful for quantum information processing.

PACS. 03.67.-a Quantum information – 32.80.Lg Mechanical effects of light on atoms, molecules, and ions – 42.50.-p Quantum optics

1 Introduction

Both cavity QED and ion traps are promising systems of quantum information processing. Experimentally, entanglement and simple quantum gates have been achieved in both systems [1]. Nevertheless, the combination of cavity QED with an ion trap seems to be more interesting due to the possibility of entanglement of more quantum degrees of freedom. Moreover, as atomic ions are fixed in the cavity, quantum gates on them can be carried out more accurately. Recently, much effort has been paid on the combinatory system — trapped ions embedded in a microcavity — for quantum information processing [2,3]. It is widely believed that such an ion-trap-cavity system is a promising candidate for building quantum networks involving cavity QED set-ups [4].

The present work will focus on generating entangled states in an ion-trap-cavity system in the strong excitation regime (SER), in which the ion-laser coupling constant is much larger than the trap frequency. We noticed that SER has been investigated in ion traps to rapidly prepare Schrödinger cat states and to fasten the quantum gating [5,6]. In contrast to the week excitation regime where the ion-trap interaction can be concisely described by Jaynes-Cummings model [7], the situation in SER is very complicated [8]. But as the large Rabi frequency can greatly reduce the implementation time, the schemes in SER are useful and important in view of decoherence. In contrast, we will show in this paper that, if SER is employed in an ion-trap-cavity system, the large Rabi frequency will not be helpful for accelerating the generation of entangled states. But we will enjoy the generation of the robust entangled states against decoherence due to the cavity decay and heating of vibrational states of the ions. Moreover, different from former work by means of large detunings [2], we only employ a laser beam with the frequency of carrier transition to radiate the ions. By considering current experimental situation, our specific discussion will be only made for two-ion case.

2 Effective Hamiltonian

We will start from the Hamiltonian presented in [3]. Let us first consider a general case. i.e., N identical cold ions confined in the ion trap which itself is embedded in a microcavity. We assume that the cavity mode, together with the radiation of a laser, couples to the internal and vibrational states of the ions. The Hamiltonian in units of $\hbar = 1$ can be generally written as follows

$$\begin{aligned} H = &\sum_{j=1}^{N} \omega_0 \sigma_z^j + \nu a^\dagger a + \omega_c b^\dagger b \\ &+ \frac{\Omega}{2} \sum_{j=1}^{N} \left[\sigma_+^j e^{i[\eta_L (a^\dagger + a) - \omega_L t]} + \text{h.c.} \right] \\ &+ \frac{g}{2} \sum_{j=1}^{N} \left(\sigma_+^j + \sigma_-^j \right) \left(b^\dagger + b \right) \sin\left[\eta_c \left(a^\dagger + a \right) + \phi \right] \end{aligned} \quad (1)$$

[a] *Current address*: Department of Mathematical Physics, National University of Ireland, Maynooth, Co. Kildare, Ireland. e-mail: mfeng@thphys.may.ie

where ω_0 is the frequency of atomic resonance transition. ω_c and ω_L are frequencies of the cavity mode and the laser respectively. $b^\dagger$, b and $a^\dagger$, a are respectively creation and annihilation operators of photons of the cavity and phonons of the trap. Ω and g are the coupling constants proportional to the ion-laser and ion-cavity interaction respectively. η_L and η_c are respectively Lamb-Dicke parameters with respect to the radiation of the laser and the cavity. $\sigma_+^j = |e\rangle_j\langle g|$, $\sigma_-^j = |g\rangle_j\langle e|$ and $\sigma_z^j = (|e\rangle_j\langle e| - |g\rangle_j\langle g|)/2$. ϕ accounts for the relative position of motional state of the ion to the standing wave of the quantized cavity field. Within the Lamb-Dicke limit, i.e. $\eta_L \ll 1$ and $\eta_c \ll 1$, $\sin[\eta_c(a^\dagger + a) + \phi] \approx \eta_c(a^\dagger + a)\cos\phi + \sin\phi$. Performing a unitary operator defined as $U = \exp[-i\omega_L(\sum_{j=1}^N \sigma_z^j + b^\dagger b + a^\dagger a)t]$ yields

$$\begin{aligned} H_1 = {} & \Delta_a a^\dagger a + \Delta_b b^\dagger b \\ & + \frac{\Omega}{2}\sum_{j=1}^{N}\Big\{\sigma_+^j\left[1 + i\eta_L\left(a^\dagger e^{i\omega_L t} + a e^{-i\omega_L t}\right)\right] \\ & + \sigma_-^j\left[1 - i\eta_L\left(a^\dagger e^{i\omega_L t} + a e^{-i\omega_L t}\right)\right]\Big\} \\ & + \frac{g}{2}\sum_{j=1}^{N}\left(\sigma_+^j b + \sigma_-^j b^\dagger\right)\left[\eta_c\left(a^\dagger e^{i\omega_L t} + a e^{-i\omega_L t}\right)\cos\phi + \sin\phi\right] \end{aligned} \tag{2}$$

where we have assumed $\omega_L = \omega_0$, i.e. the carrier transition case, $\Delta_a = \nu - \omega_L$, $\Delta_b = \omega_c - \omega_L$ and rotating-wave approximation has been used. Further rotation of the system with respect to $\exp\{-i(\Delta_a a^\dagger a + \Delta_b b^\dagger b)t\}$ yields

$$\begin{aligned} H_2 = {} & \Omega\sum_{j=1}^{N} S_z^j + \frac{1}{2}i\eta\Omega\sum_{j=1}^{N}\left(S_+^j - S_-^j\right)\left(a^\dagger e^{i\nu t} + a e^{-i\nu t}\right) \\ & + \frac{g}{2}\sum_{j=1}^{N}\Big[S_z^j\left(b e^{-i\Delta_b t} + b^\dagger e^{i\Delta_b t}\right) \\ & + \frac{1}{2}\left(S_+^j - S_-^j\right)\left(b e^{-i\Delta_b t} - b^\dagger e^{i\Delta_b t}\right)\Big] \\ & \times\left[\eta_c\left(a^\dagger e^{i\nu t} + a e^{-i\nu t}\right)\cos\phi + \sin\phi\right] \end{aligned} \tag{3}$$

where we have introduced $S_+^j = |+\rangle_j\langle -|$, $S_-^j = |-\rangle_j\langle +|$ and $S_z^j = (|+\rangle_j\langle +| - |-\rangle_j\langle -|)/2$ with $|+\rangle_j = (|g\rangle_j + |e\rangle_j)/\sqrt{2}$ and $|-\rangle_j = (|g\rangle_j - |e\rangle_j)/\sqrt{2}$ [9].

Our aim is to have a subspace involving only the internal levels of the ions. To this end, we have to make a further rotation with respect to Ω, which results in

$$\begin{aligned} H_3 = {} & \frac{1}{2}i\eta_L\Omega\sum_{j=1}^{N}\left(S_+^j e^{i\Omega t} - S_-^j e^{-i\Omega t}\right)\left(a^\dagger e^{i\nu t} + a e^{-i\nu t}\right) \\ & + \frac{g}{2}\sum_{j=1}^{N}\Big[S_z^j\left(b e^{-i\Delta_b t} + b^\dagger e^{i\Delta_b t}\right) \\ & + \frac{1}{2}\left(S_+^j e^{i\Omega t} - S_-^j e^{-i\Omega t}\right)\left(b e^{-i\Delta_b t} - b^\dagger e^{i\Delta_b t}\right)\Big] \\ & \times\left[\eta_c\left(a^\dagger e^{i\nu t} + a e^{-i\nu t}\right)\cos\phi + \sin\phi\right]. \end{aligned} \tag{4}$$

Consider the strong excitation regime, i.e., $\Omega \gg \nu$. As long as $\eta_L\Omega \le g$ and $\Omega \pm \nu \gg |\Omega - \Delta_b|$, the first term in the right hand side of equation (4) can be neglected due to the fast oscillation produced by large detuning. Accordingly, if $\phi = \pi/2$ and $|\Omega - \Delta_b| = \delta \ll |\Delta_b| < |\Omega + \Delta_b|$, we can single out

$$H_4 = \frac{g}{4}\sum_{j=1}^{N}(S_+^j e^{i\delta t} b + S_-^j e^{-i\delta t} b^\dagger) \tag{5}$$

from above Hamiltonian. More specifically, if $\delta \gg g\sqrt{\bar{n}_b + 1}/4$ with $\bar{n}_b$ the mean photon number in the cavity, there would be no energy exchange between internal states of the ions and the cavity state. As a result, equation (5) turns to an effective Hamiltonian [10]

$$H_5 = \frac{\tilde{\Omega}}{2}\left[\sum_{j=1}^{N}(|+\rangle_j\langle +|bb^\dagger - |-\rangle_j\langle -|b^\dagger b) + \sum_{j,k=1,j\neq k}^{N}(|+-\rangle_{jk}\langle -+| + \text{h.c.})\right] \tag{6}$$

where

$$\begin{aligned} \tilde{\Omega}/2 = {} & \\ & \langle +-n_b|H_4|++n_b-1\rangle\langle ++n_b-1|H_4|-+n_b\rangle/\delta \\ & - \langle +-n_b|H_4|--n_b+1\rangle\langle --n_b+1|H_4|-+n_b\rangle/\delta \\ & = -g^2/(16\delta), \end{aligned}$$

with $|\ \rangle$ being the product of internal states of the ions j and k, and the cavity state. Equation (6) is a typical XY Hamiltonian, which has been extensively investigated. Based on it, the multipartite entangled states can be built among the ions, without the involvement of states of the cavity and the ions' vibration. If we encode logical qubits in the state-space of pairs of adjacent ions as $|0_L\rangle_i := |-+\rangle_{i,i+1}$ and $|1_L\rangle_i := |+-\rangle_{i,i+1}$, universal quantum computing [11] could be achieved just by time-dependent control of the XY Hamiltonian with nearest-neighbor and next-nearest-neighbor interactions, without the necessity of single-qubit operations.

3 Bipartite entangled states

In what follows, however, our discussion will be only focused on the two-ion case due to following reasons: (i) it is experimentally challenging to confine more than one atomic ions in the ion-trap-cavity system. Current experimental effort is still paid on a single ion in such a combinatory system [12]; (ii) a quantum network is composed of many nodes with each node involving few qubits. The two ions in a cavity is a working node with minimum numbers of qubits; (iii) Bell states play important and fundamental roles in quantum information processing. So the investigation of Bell states in this system will be by no doubt useful and practical. Since the first term of equation (6), which is associated with the photon dependent Stark shift, commutes with the second term, we can easily obtain the time evolution of the system of two ions A and B as

$$|+-\rangle_{AB} \to e^{-i\tilde{\Omega}t/2}\left[\cos\left(\frac{\tilde{\Omega}}{2}t\right)|+-\rangle - i\sin\left(\frac{\tilde{\Omega}}{2}t\right)|-+\rangle\right]_{AB}$$

and

$$|-+\rangle_{AB} \to e^{-i\tilde{\Omega}t/2}\left[\cos\left(\frac{\tilde{\Omega}}{2}t\right)|-+\rangle - i\sin\left(\frac{\tilde{\Omega}}{2}t\right)|+-\rangle\right]_{AB}.$$

Returning to the Schrödinger representation and rewriting above time evolution with states $|g\rangle$ and $|e\rangle$ yield

$$\begin{aligned}|gg\rangle \to &\frac{1}{4}\left(e^{-i\Omega t-i\tilde{\Omega}t}+e^{i\Omega t}+2e^{-i\tilde{\Omega}t}\right)e^{i\omega_0 t}|gg\rangle\\ &+\frac{1}{4}(e^{-i\Omega t-i\tilde{\Omega}t}+e^{i\Omega t}-2e^{-i\tilde{\Omega}t})e^{-i\omega_0 t}|ee\rangle\\ &+\frac{1}{4}(e^{-i\Omega t-i\tilde{\Omega}t}-e^{i\Omega t})(|eg\rangle+|ge\rangle),\end{aligned} \tag{7}$$

$$\begin{aligned}|ee\rangle \to &\frac{1}{4}\left(e^{-i\Omega t-i\tilde{\Omega}t}+e^{i\Omega t}-2e^{-i\tilde{\Omega}t}\right)e^{i\omega_0 t}|gg\rangle\\ &+\frac{1}{4}\left(e^{-i\Omega t-i\tilde{\Omega}t}+e^{i\Omega t}+2e^{-i\tilde{\Omega}t}\right)e^{-i\omega_0 t}|ee\rangle\\ &+\frac{1}{4}\left(e^{-i\Omega t-i\tilde{\Omega}t}-e^{i\Omega t})(|eg\rangle+|ge\rangle\right),\end{aligned} \tag{8}$$

$$\begin{aligned}|eg\rangle \to &\frac{1}{4}\left(e^{-i\Omega t-i\tilde{\Omega}t}-e^{i\Omega t}\right)\left(e^{i\omega_0 t}|gg\rangle+e^{-i\omega_0 t}|ee\rangle\right)\\ &+\frac{1}{4}\left(e^{-i\Omega t-i\tilde{\Omega}t}+e^{i\Omega t}\right)(|eg\rangle+|ge\rangle)\\ &+\frac{1}{2}(|eg\rangle-|ge\rangle),\end{aligned} \tag{9}$$

$$\begin{aligned}|ge\rangle \to &\frac{1}{4}\left(e^{-i\Omega t-i\tilde{\Omega}t}-e^{i\Omega t}\right)\left(e^{i\omega_0 t}|gg\rangle+e^{-i\omega_0 t}|ee\rangle\right)\\ &+\frac{1}{4}\left(e^{-i\Omega t-i\tilde{\Omega}t}+e^{i\Omega t}\right)(|eg\rangle+|ge\rangle)\\ &+\frac{1}{2}(|ge\rangle-|eg\rangle),\end{aligned} \tag{10}$$

where for simplicity we have assumed the cavity to be vacuum, i.e., $\bar{n}_b=0$ and dropped the subscripts. The global phase $\exp\{-i\bar{n}_a\omega_0 t\}$ is also neglected in above equations of time evolution. It is evident that above time evolutions lead to the states much more complicated than those achievable in ion traps or cavity QED by large detunings [10,13]. To get more physical insight into our system, however, let us first check the case at $t=\pi/\tilde{\Omega}=2\pi(k+1/4)/\Omega$ $(k=0,1,...)$, in which we have entangled states

$$(1/2)[(i-1)e^{i\omega_0 t}|gg\rangle+(i+1)e^{-i\omega_0 t}|ee\rangle],$$
$$(1/2)[(i+1)e^{i\omega_0 t}|gg\rangle+(i-1)e^{-i\omega_0 t}|ee\rangle],$$
$$(1/2)[(i+1)|eg\rangle+(i-1)|ge\rangle],$$
$$(1/2)[(i-1)|eg\rangle+(i+1)|ge\rangle],$$

from initial states $|gg\rangle$, $|ee\rangle$, $|eg\rangle$, $|ge\rangle$, respectively. These states would be useful in quantum information processing although they are not maximally entangled [14]. If we further assume the initial states of the ions to be Bell states [15], equations (7–10) would yield,

$$\begin{aligned}\Psi^+ = &\frac{1}{\sqrt{2}}(|gg\rangle+|ee\rangle)\to\\ &\frac{1}{2\sqrt{2}}e^{i\Omega t}\left[e^{-i(2\Omega+\tilde{\Omega})t}+1\right]\left(e^{i\omega_0 t}|gg\rangle+e^{-i\omega_0 t}|ee\rangle\right)\\ &+\frac{1}{2\sqrt{2}}e^{i\Omega t}\left[e^{-i(2\Omega+\tilde{\Omega})t}-1\right](|eg\rangle+|ge\rangle),\end{aligned} \tag{11}$$

$$\begin{aligned}\Phi^+ = &\frac{1}{\sqrt{2}}(|ge\rangle+|eg\rangle)\to\\ &\frac{1}{2\sqrt{2}}e^{i\Omega t}\left[e^{-i(2\Omega+\tilde{\Omega})t}-1\right]\left(e^{i\omega_0 t}|gg\rangle+e^{-i\omega_0 t}|ee\rangle\right)\\ &+\frac{1}{2\sqrt{2}}e^{i\Omega t}\left[e^{-i(2\Omega+\tilde{\Omega})t}+1\right](|eg\rangle+|ge\rangle),\end{aligned} \tag{12}$$

$$\Psi^- = \frac{1}{\sqrt{2}}(|gg\rangle-|ee\rangle)\to\frac{1}{\sqrt{2}}e^{-i\tilde{\Omega}t}\left(e^{i\omega_0 t}|gg\rangle-e^{-i\omega_0 t}|ee\rangle\right), \tag{13}$$

$$\Phi^- = \frac{1}{\sqrt{2}}(|ge\rangle-|eg\rangle)\to\frac{1}{\sqrt{2}}(|ge\rangle-|eg\rangle). \tag{14}$$

4 Discussion

Some unique characteristics under our model can be found from above four equations.

(1) Starting from Ψ^+ or Φ^+, we can reach the entangled states involving $|gg\rangle$, $|ee\rangle$, $|ge\rangle$ and $|eg\rangle$ simultaneously. In contrast, in the well-known quantum computing work with hot trapped ions [10], $\Psi^\pm$ and $\Phi^\pm$ are produced independently from different initial conditions: If the two ions are initially in $|gg\rangle$ or $|ee\rangle$, we can only obtain $\Psi^\pm$ at a certain time of the time evolution due to radiation of the largely detuned lasers. The initial states $|ge\rangle$ and $|eg\rangle$ would only yield $\Phi^\pm$. Since the generation of $\Psi^\pm$ needs external energy source, such as laser beams, in cavity QED scheme with large detuning between the cavity light and atomic resonance, only $\Phi^\pm$ can be generated [13]. In contrast, our system is composed of the ion trap and cavity QED. Although we have turned it into a formally cavity QED system, the energy from the laser beam makes it possible to have $\Psi^\pm$ in the system.

(2) The combination of the ion trap and cavity QED is not simply equivalent to the summation of the results

appeared separately in the component systems. Equations (11, 12) show that, Ψ^+ and Φ^+ can be interchanged if we neglect the relative phase between $|gg\rangle$ and $|ee\rangle$, which can be canceled by local operation based on the knowledge of ω_0 and t. It is also shown in equation (14) that Φ^- plays like a dark state, which is very stable and would be useful for quantum communication, e.g. teleportation.

(3) The system under consideration is more complicated than the system with only an ion trap or a cavity due to more quantum degrees of freedom involved [3,16]. Although there are many possible solutions, to gain the physics we are interested in, we only focused in this work on the generation of entangled internal states of the ions, which is based on equation (5). Equation (5) is important for quantum network because the photons, which are employed to connect different nodes of the network, obtain quantum information from the cavity mode. If we change the laser frequency to make $\delta = 0$ in equation (5), we have the typical Jaynes-Cummings interaction between the ions and the cavity mode [17]. Therefore, by adjusting the laser frequency, we can either have the possibility of generating entangled internal states of the ions, which is the prerequisite of robust quantum computing, or have the possibility of exchanging information in ion's internal states with the cavity mode.

(4) Different from in ion traps [5,6], the large Rabi frequency Ω cannot speed up the generation of entangled states in our combinatory setup. On the contrary, the difference between Ω and Δ_b should be large enough to reduce susceptibility of population in the cavity mode, which, to some extent, prolongs the implementation time. Nevertheless, the remarkable feature of our scheme is the generation of entangled states in the subspace involving only the internal states of the ions. Since the trap degrees of freedom are decoupled from our model, and the cavity mode is only virtually excited, the generated entangled state would be robust. We will give a quantitative discussion for this point below.

It is very difficult to realize the SER experimentally, even only in ion traps. To achieve the effective Hamiltonian equation (6) of our scheme, we have to have some parameter values beyond the reach of current experimental technique, for example, $\Delta_b = 1.1$ MHz, $\Omega = 1.15$ MHz, $\nu = 0.1$ MHz, $\eta_L = 0.01$ and $g = 0.02$ MHz. To test the validity of the approximations we made from equation (3) to equation (6), we made some numerical simulation, as shown in Figures 1 and 2, where for simplicity we only focus our calculation on the internal states and only present the time evolution of population regarding $|+-\rangle$ for clarity. The figures show that the effective Hamiltonian equation (6) is valid with above parameter values. The figures also tell us that if we actually reach the SER, a very small η_L is essential to our scheme. With currently achievable number $\eta_L = 0.1$ [18], we cannot effectively decouple the vibrational degrees of freedom of the ions from their internal states.

Moreover, the condition $\delta \gg g\sqrt{\bar{n}_b + 1}/4$ is very important to our approximation, which strongly restricts the speed of our gate implementation. Besides Figure 1, we can also define $R = g^2/(16\delta^2)$, the possible excitation of the intermediate state [18], to check this point. For the case of curves A in Figure 1, $|\tilde{\Omega}/2|$ should be 500 Hz, and $R = 1\%$. This implies that the fidelity of our gating is 99%. But If we want to speed up our gating by enlarging $g/4\delta$ to be 0.5, as considered in curves B, the gating fidelity would be only 75%.

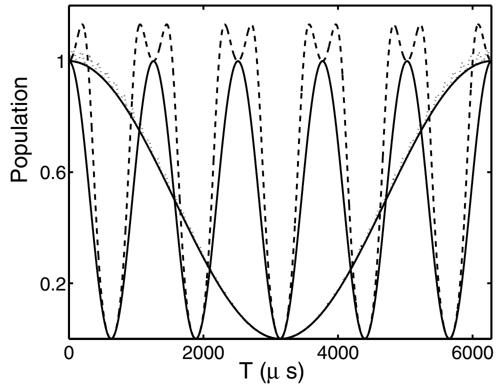

Fig. 1. Time evolution of population in the state $|+-\rangle$, where $\bar{n}_b = 0$ and two cases are considered. Curves A (i.e. the ones with one period) correspond to $\Delta_b = 1.1$ MHz, $\Omega = 1.15$ MHz, $\nu = 0.1$ MHz, $\eta_L = 0.01$ and $g/4\delta = 0.1$ MHz, in which the solid curve is from equation (6), and the dotted curve is from equation (3). Curves B (i.e. the ones with 5 periods) are drawn with $\Delta_b = 1.1$ MHz, $\Omega = 1.15$ MHz, $\nu = 0.1$ MHz, and $g/4\delta = 0.5$ MHz, where solid and dashed curves are from equation (6) and equation (3) respectively.

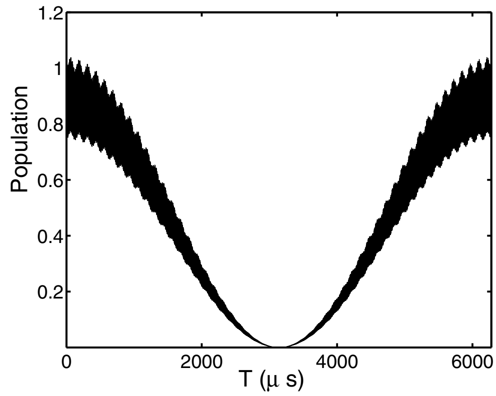

Fig. 2. Time evolution of population in the state $|+-\rangle$, where $\eta_L = 0.1$ and other parameters are the same as in curves A in Figure 1.

Let us briefly discuss about decoherence. As we mentioned above, the SER case in our combinatory setup cannot reduce the gating time. As a result, we have to turn the system into an effective form (i.e., Eq. (6)) by decoupling cavity mode and the vibrational states of the ions. However, even if equation (6) can be achieved, we should still pay attention to the cavity decay because the cavity mode plays the role of date bus in our scheme. More strictly speaking, even if we keep the cavity in a vacuum state throughout our scheme, any unpredictable fluctuation of the cavity mode will probably affect our scheme.

In current ion-trap-cavity experiment, the cavity decay rate is $2\pi \times 102$ kHz [12]. By considering the excitation probability $R = 1\%$, the decay rate in our case is $2\pi \times 1020$ Hz, which is still larger than $|\tilde{\Omega}/2|$ (= 500 Hz). This means that we have to much improve the current cavity quality in order to achieve our scheme. Another important source of decoherence is the spontaneous emission from $|e\rangle$. But this decoherence can be neglected in our consideration if we employ the metastable state to be the state $|e\rangle$, as done in [12].

Since the Rabi frequency Ω is much larger than the effective Rabi frequency $\tilde{\Omega}$, we should also pay attention to the possible influence from the fluctuation of Ω. From equations (4, 5), we know that the validity of our scheme depends on the fast oscillating terms being effectively averaged out. So although Ω is much larger than g, our rotating-wave approximation will be available as long as $\eta_L \Omega/2 \ll \delta + \Delta_b - \nu$ [19]. This requires $\eta_L \ll 1.83$ for the case of curves A in Figure 1, which fully meets the Lamb-Dicke requirement $\eta \ll 1$. Therefore in the Lamb-Dicke assumption, the influence from the fluctuation of Ω can be neglected in our scheme.

5 Conclusion

Although the realization of SER is experimentally challenging, we noted a possible way to achieve SER in ion traps [5]: we first cool the ions within the Lamb-Dicke limit and under the weak excitation regime. Then we decrease the trap frequency by opening the trap adiabatically so that the ratio of the Rabi frequency to the trap frequency is increased to a large number. It is evident that this method can be transplanted to our combinatory set-up.

In summary, a scheme has been proposed for generating entangled states of the ions confined in trap-cavity combinatory setup in SER, without the involvement of the cavity state and vibrational state of the ions. The unique characteristics of the bipartite entangled states have been discussed in our system. To our knowledge, it is the first quantum information processing scheme of trap-cavity combinatory setup in SER. Although the SER is very hard to reach with current experimental technique, our scheme would be useful in future experiments of ion-trap-cavity setup both for the test of quantum mechanics and for quantum information processing.

The author thanks Graham Kells for his assistance in numerical simulation. The work is supported by Chinese Academy of Sciences.

References

1. A. Rauschenbeutel, G. Nogues, S. Osnaghi, P. Brune, J. Raimond, S. Haroche, Science **288**, 2024 (2000); C. Monroe, D.M. Meekhof, B.E. King, D.J. Wineland, Science **272**, 1131 (1996)
2. J. Pachos, H. Walther, Phys. Rev. Lett. **89**, 187903 (2002); E. Jane, M.B. Plenio, D. Jonathan, Phys. Rev. A **65**, 050302 (2002); M. Feng, Phys. Rev. A **65**, 044304 (2002)
3. M. Feng, X. Wang, J. Opt. B: Quant. Semiclass. Opt. **4**, 283 (2002)
4. S. Bose, P.L. Knight, M.B. Plenio, V. Vegral, Phys. Rev. Lett. **83**, 5158 (1999); S.J. van Enk, J.I. Cirac, P. Zoller, Science **279**, 205 (1998)
5. J.F. Poyators, J.I. Cirac, R. Blatt, P. Zoller, Phys. Rev. A **54**, 1532 (1996)
6. S. Zheng, X.W. Zhu, M. Feng, Phys. Rev. A **62**, 033807 (2000); J.F. Poyatos, J.I. Cirac, P. Zoller, Phys. Rev. Lett. **81**, 1322 (1998)
7. J.I. Cirac, A.S. Parkins, R. Blatt, P. Zoller, Adv. At. Mol. Opt. Phys. **37**, 237 (1996)
8. H. Zeng, F. Lin, Y. Wang, Y. Segawa, Phys. Rev. A **59**, 4589 (1999); M. Feng, Eur. Phys. J. D **18**, 371 (2002)
9. D. Jonathan, M.B. Plenio, P.L. Knight, Phys. Rev. A **62**, 042307 (2000); D. Jonathan, M.B. Plenio, Phys. Rev. Lett. **87**, 127901 (2001)
10. A. Sørensen, K. Mølmer, Phys. Rev. Lett. **82**, 1971 (1999)
11. D.A. Lidar, L.-A. Wu, Phys. Rev. Lett. **88**, 017905 (2002)
12. A.B. Mundt, A. Kreuter, C. Becher, D. Leibfried, J. Eschner, F. Schmidt-Kaler, R. Blatt, Phys. Rev. Lett. **89**, 103001 (2002)
13. S. Zheng, G. Guo, Phys. Rev. Lett. **85**, 2392 (2000)
14. Z. Kurucz, M. Koniorczyk, J. Janszky, Fortschr. Phys. **49**, 1019 (2001)
15. With $t = \pi/\tilde{\Omega} = 2\pi(n+1/4)/\Omega = 2\pi(m\pm1/8)/\omega_0$ ($m, n = 0, 1, ...$), we can have $\Psi^{\pm} = (|gg\rangle \pm |ee\rangle)/\sqrt{2}$ from initial states $|gg\rangle$ and $|ee\rangle$ respectively [see Eqs. (7, 8)]. However, we have not yet found how to produce $\Phi^{\pm} = (|ge\rangle \pm |eg\rangle)/\sqrt{2}$ by means of equations (9, 10). Nevertheless, we argue that we are able to produce $\Phi^{\pm}$ by other means, for example, preparing them initially in an ion trap. Then we change laser frequency and add a cavity field to have equation (5), or move the ion from the ion trap to our trap-cavity combinatory setup. The latter is similar to the multi-trap proposal published in Nature **417**, 709 (2002), and will be achievable according to current experimental progress [see, D. Leibfried, J. Phys. B **36**, 599 (2002)]. On the other hand, we can consider the discussion based on the Bell states as good examples, most of whose features can be generalized to the cases of non-maximal entangled states. As we will see below, most features pointed out in next section for the time evolution of Bell states are also suitable for non-maximal entangled states
16. See `http://heart-c704.uibk.ac.at/cavity_qed.html`
17. In this case, the laser frequency is smaller than the former one by δ. So the deduction from equation (2) should be modified due to appearance of the detuning to ω_0. By straightforward algebra, we can obtain $H = g/4 \sum_{j=1}^{N} (S_+^j b + S_-^j b^\dagger)$
18. C.A. Sackett, D. Kielpinski, B.E. King, C. Langer, V. Meyer, C.J. Myatt, M. Rowe, Q.A. Turchette, W.M. Itano, D.J. Wineland, C. Monroe, Nature **404**, 256 (2000)
19. B.W. Shore, P.L. Knight, J. Mod. Opt. **40**, 1195 (1993)